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Empirical Formulae Evaluation for Hydraulic Conductivity Determination Based on Grain Size Analysis

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Seven empirical formulae to calculate hydraulic conductivity using grain size distribution of unconsolidated aquifer materials have been evaluated in this study. Four soil samples extracted from an injection or recharge well during borehole drilling aimed at recharging the ground water through rainwater harvesting in Lahore Pakistan. The sample testing and grading analysis were done in CEWRE, UET Lahore soil laboratory. Results showed that out of seven empirical formulae three formulae (Kozeny-Carman, Hazen and Breyer) reliably estimated hydraulic conductivities of the various soil samples well within the known ranges while the others four formulae Slitcher, Terzaghi, USBR, Alyamani & Sen methods underestimated the results as compared to constant head method results for all samples. Kozeny-Carman Equation proved to be the best estimator of most samples analyzed, and maybe, even for a wide range of other soil types. This best estimation is followed by Hazen and Breyer formula after Kozeny-Carman formula. Most importantly, all these empirical formulae are to be used strictly within their domains of applicability.

Keywords: Hydraulic Conductivity, Empirical formulae, Grain-size analysis, constant head permeameter

INTRODUCTION

The hydraulic conductivity (K) is a hydro-geologic property of the medium which refers to the ease with which a fluid can flow through the medium. It depends upon the porous medium and flowing fluid. Mathematically hydraulic conductivity is defined as $K = {k\rho g}/{\mu}$ Where, k=intrinsic permeability of porous medium and ρ and μ are density and dynamic viscosity of fluid respectively.

Hydraulic conductivity is a direct function of average grain size distribution of granular porous media (Freeze and Cherry 1979). As the average grain size decreases from sand to clay, therefore $K_{sand}>K_{silt}>K_{clay}$. An increased emphasis is nowadays on environmental aspects of ground water problems, so a complete characterization of hydraulic conductivity in term of average, spread and distribution in the flow domain is essential for dealing with modeling the soil water flow in saturated and unsaturated zones and water-soluble contamination transport. The saturated hydraulic conductivity of a soil is very important parameter in the field of ground water hydrology for designing and construction of earth fill, rock fill dams and levee. Furthermore, it has paramount importance for the determination of seepage losses, hydraulic structures, settlements and stability analysis, and other geotechnical problems (Boadu 2000).

Hydraulic conductivity determination can be done by different techniques such as field methods (pumping test of wells, auger hole test and tracer test), laboratory methods (constant head and falling head permeameters) and calculations from empirical formulae (Todd and Mays 2005).However the field methods are limited for accurate estimation of hydraulic conductivity due to aquifer geometry and hydraulic boundaries precise knowledge (Uma *et al.*, 1989) as well as the cost of well construction and operations. The laboratory tests provide formidable problems in the sense of true representative samples. Both methods are usually restricted to hydraulic conductivity determined near the ground surface and as such may not be representative of the aquifer materials. Alternatively, empirical formulae for estimating the hydraulic conductivity based on grain-size distribution characteristics have been developed and used to overcome these problems. Grain- size distribution methods are comparably less expensive and do not depend on the geometry and hydraulic boundaries of the aquifer.

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Soil is often made up of grains of many different sizes and textures, the sizes are measured in terms of grain size distribution based on the diameters of pores rather than those of grains. The pore size distribution is very difficult to determine so the potential alternate is the grain size distribution as a substitute which is easy to measure and used for the approximation of hydraulic properties and estimation of hydraulic conductivity (Cirpka 2003). Consequently, groundwater professionals have tried for decades to relate hydraulic conductivity to grain size. The tasks appear rather straight forward, but it found that this correlation is not easily established (Pinder and Celia 2006). Several formulae have been established by many researchers and scientists based on experimental work using the hydraulic conductivity and grain size relationship, such as Hazen (1892), Kozeny (1927), Carman (1937, 1956), Terzaghi and Peck (1964), Shepherd (1989), Alyamani and Sen (1993). The applicability of these formulae depends on the type of soil in which hydraulic conductivity is to be estimated.

The objective of this study is to evaluate the applicability and reliability of some of the commonly used empirical formulae for the determination of hydraulic conductivity of unconsolidated soil materials.

1.1. Commonly Established Empirical Formulae

Mathematically, the Hydraulic conductivity (K) = ${k\rho g}/{\mu}$ where

k intrinsic permeability which only depends on the properties of soil while hydraulic conductivity (permeability) is the property of both, the soil and the fluid. So in the established empirical formulae the different scientist defined the intrinsic permeability (k).

(1) **Hazen Formula:** It was widely used for the estimation of hydraulic conductivity of uniformly graded soils ranges from fine sand to gravel of diameter 0.1 to 3 mm respectively. This formula only depends on the effective size of grains as given below.

$$K = {\rho g}/{\mu} \times 6 \times 10^{-4} \times [1 + 10(n - 0.26)](d_{10})^2$$

(2) **Kozeny-Carman Equation:** The KC equation is widely used and accepted for hydraulic conductivity estimation because it depends on both the effective grain size and porosity (number of pores) of the porous media as given below.

$$K = {\rho g}/{\mu} \times 8.3 \times 10^{-3} \times \left[\frac{n^3}{(1-n)^2}\right](d_{10})^2$$

(3) **Breyer Formula:**

$$K = {\rho g}/{\mu} \times 6 \times 10^{-4} \times \log\frac{500}{U} \times (d_{10})^2$$

This formula does not consider porosity and is often most useful for materials with heterogeneous distributions and poorly sorted grains with a uniformity coefficient between 1 and 20, and effective grain size between 0.06mm and 0.6mm.

(4) **Slitcher Formula:**

$$K = {\rho g}/{\mu} \times 1 \times 10^{-2} \times n^{3.287} \times (d_{10})^2$$

This formula is most applicable for grain-size between 0.01mm and 5mm.

(5) **Terzaghi Formula:**

$$K = {\rho g}/{\mu} \times C_t \times \left(\frac{n-0.13}{\sqrt[3]{1-n}}\right)^2 \times (d_{10})^2$$

Where the C_t = sorting coefficient and 6.1 ×10-3< C_t< 10.7 × 10-3. In this study, an average value of C_t (8.4 × 10-3) is used. Terzaghi formula is most applicable for large-grain sand (Cheng and Chen 2007).

(6) **USBR Formula:**

$$K = {\rho g}/{\mu} \times 4.8 \times 10^{-4} \times (d_{20})^{2.3}$$

United State Bureau of Reclamation (USBR) formula, estimates hydraulic conductivity from the effective grain size (d_{20}). This formula is suitable for medium-grain sand with a uniformity coefficient less than 5 (Cheng and Chen 2007).

(7) **Alyamani & Sen Formula:**

$$K = 1300 \times [I_0 + 0.025(d_{50} - d_{10})]^2$$

Where K is the hydraulic conductivity (m/day), I_o is the intercept (in mm) of the line formed by d_{50} and d_{10} with the grain-size axis, d_{10} is the effective grain diameter (mm), and d_{50} is the median grain diameter (mm).

2. MATERIALS AND METHODS

For the evaluation of empirical formulae, the results of hydraulic conductivity were compared with the results achieved by using laboratory methods (constant head method). The step by step procedure applied for hydraulic conductivity determination using empirical formulae and constant head methods are given below.

2.1. Samples Test

Four different soil samples were taken from an injection or recharge well during borehole drilling aimed at recharging the ground water through rainwater harvesting in Lahore Pakistan. Samples were collected and taken to the Center of Excellence in Water Resources Engineering, University of Engineering and Technology soil testing laboratory for further analysis. From the laboratory, the samples were treated and tested for grain size distribution according to the standard procedures of BS1377. The samples were tested by the method of dry sieve analysis using a series of sorted BS sieves.

2.2. Grain Size Distribution and Samples Classification

The grain size distribution of four samples was done as shown in table 1 below

Table 1: summary of soil particle size distribution tests

Opening (mm)	0.84	0.71	0.50	0.35	0.21	0.18	0.11	0.06	Pan
Percent Passing									
Sample 1	99.90	99.83	97.00	86	48.63	33.37	6.60	2.60	0
Sample 2	81.03	66.18	30.49	13.67	7.93	6.69	4.19	4.17	0
Sample 3	100	98.56	67.98	34.23	12.67	5.98	4.20	2.45	0
Sample 4	98.36	96.53	57.90	30.63	11.00	7.86	5.40	3.21	0
Sample 4	98.36	96.53	57.90	30.63	11.00	7.86	5.40	3.21	0

Further the grain size distribution curves for each sample were plotted (particle diameter on x-axis and percent passing on y-axis) in the semi-log graph as shown in Figure.1

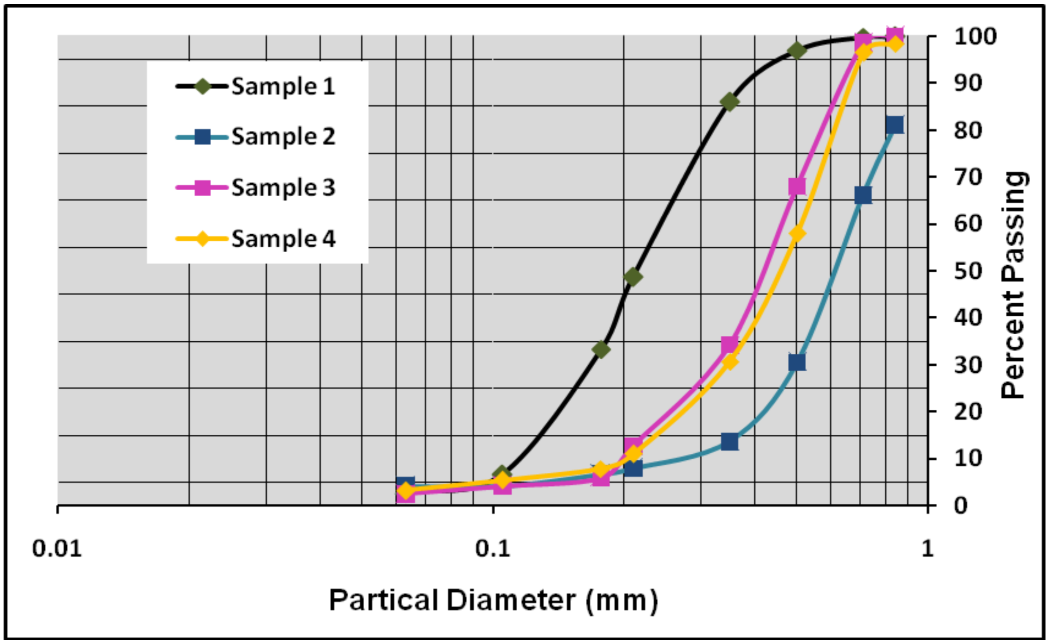


Figure 1: Grain size distribution curves for soil samples

From the grain-size distribution curves, soil samples were classified according to particle size using a standard British Soil Classification System, the soils are classified into named basic soil-type groups according to size, and the groups further divided into coarse, medium and fine sub-groups. As the distribution curves indicate the basic soil type is the sandy soil and further sub-types were done using the following range as given below.

Basic Soil Type	Sub-Type	Range of Diameter (mm)
Sandy	Fine sand	0.06 to 0.2
	Medium Sand	0.2 to 0.5
	Coarse Sand	0.5 to 2.0

According to this classification two samples (3&4) are under medium sand type while sample 2 has coarse sand type and sample 1 has a fine sand type classification.

2.3. Determination of Hydraulic Conductivity (K) from Grain Size Analysis

Using the grain-size distribution curves in the soil samples were classified, diameters of soil particles at 10%, 20% and 50% cumulative weight determined, and the coefficients of uniformity (U), intercepts and porosity values were calculated. The hydraulic conductivities were calculated using the seven empirical formulae discussed above. The results are presented in table 2 below.

Table 2: Hydraulic conductivities calculated from empirical formulae

Sample & Its Classification	d10 (mm)	d20 (mm)	d50 (mm)	d60 (mm)	U= (d60/d10)	*n=0.255(1+0.83U)	I0 (mm)	Hazen (m/d)	K-C (m/d)	Breyer (m/d)	Slitcher (m/d)	Terzaghi (m/d)	USBR (m/d)	A/S (m/d)
Sample 1 (Fine Sand)	0.12	0.15	0.22	0.25	2.08	0.43	0.063	19.62	24.27	17.43	7.50	---------	---------	5.58
Sample 2 (Coarse Sand)	0.28	0.42	0.62	0.68	2.43	0.42	0.105	102.54	117.90	92.24	37.54	65.59	---------	16.75
Sample 3 (Medium Sand)	0.21	0.26	0.42	0.47	2.24	0.42	0.09	58.99	70.56	52.68	22.11	---------	18.36	11.79
Sample 4 (Medium Sand)	0.21	0.28	0.46	0.52	2.48	0.42	0.1	57.36	65.32	51.70	20.88	---------	21.77	14.68

Key: K-C = Kozeny-Carman; A/S = Alyamani& Sen; * Vukovic and Soro (1992)

2.4. Laboratory Method for Determination of Hydraulic Conductivity

Hydraulic conductivity can be determined in a laboratory using an apparatus called permeameter. Here constant head permeameter was used which is mostly used for materials with medium to high hydraulic conductivity. In this method a constant discharge is established through the soil sample under a constant head difference between the level of constant water supply source and the discharge end of the sample. The discharge Q passing through the sample of length L and cross sectional area A under head difference of Δh is measured. The hydraulic conductivity is then calculated as:

$$K = QL/A.\Delta h$$

Where,

K = permeability (cm/sec), L= length of specimen (cm), t= average time for discharge (sec), Q= V/t (cm^3 /sec), V= volume of discharge (cm^3), A= cross sectional area of Permeameter (cm^2) = $\pi d^2/4$, where d= inner diameter of cylinder (cm), Δh = hydraulic head difference across length L, in cm of water

In the laboratory following results were obtained using standard methods of hydraulic conductivity determination through constant head permeameter.

Length of Soil Specimen, L = 5 cm

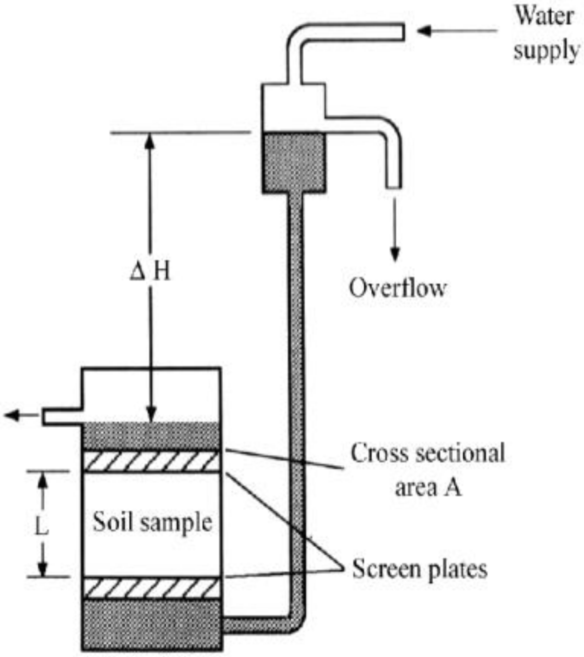
Diameter of the Soil Specimen (Permeameter), D= 5 cm

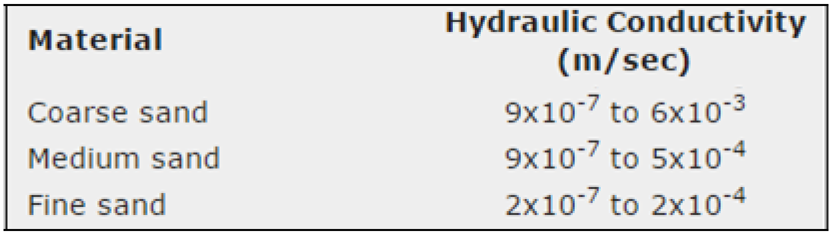
Area of specimen, A= 19.63 cm^2

Δh = hydraulic head difference = 12.7 cm

Volume of water, V = 200 cc

Sr.No	t_{av} (sec)	Q= V/t (cc/sec)	K (cm/sec)	K (m/d)
1	114	1.75	0.035	30.24
2	29	6.89	0.138	119.39
3	48	4.16	0.083	72.08
4	62	3.22	0.064	55.79





Material	Hydraulic Conductivity (m/sec)
Coarse sand	9×10^{-7} to 6×10^{-3}
Medium sand	9×10^{-7} to 5×10^{-4}
Fine sand	2×10^{-7} to 2×10^{-4}

The conductivity of four soil samples determined by constant head permeameter is in the range as given by Domenico and Schwartz 1990.

3. RESULTS AND DISCUSSIONS

The quantitative analysis of grading curves using certain geometric values known as grading characteristics, i.e. d_{10}, d_{30}, d_{60} etc. from these characteristic sizes the effective size (de = d_{10}), uniformity coefficient ($U=d_{60}/d_{10}$), and coefficient of gradation or curvature [$C^c = (d_{30})^2/(d_{60}\times d_{10})$].The soil sample classification indicated that the Sample 1- comprised 4% coarse sand, 36% medium sand and 60% fine sand, therefore classified as fine sand. Sample-2 comprised 10% fine sand, 10% medium sand and 80% coarse sand and classified as coarse sand. The sample- (3&4) have almost same configuration, i.e. 17% fine sand, 70% medium sand, 13% coarse sand, so called as medium sand. Also, all the samples showed uniform soil condition because uniformity coefficient is less than 3 and the grading curves are uniform grading curves due to the C_c ranges 0.5 to 2.0.

Hydraulic conductivities for fine and medium sand samples are not available for Terzaghi method because the formula is only suitable for coarse-grain sand. On the other hand, the conductivity value for coarse sand and fine sand is not available for USBR since the method is only relevant for medium-grain sand. For the evaluation of empirical formulae, comparative analysis was done with constant head permeameter results of all samples as shown in Figure.2

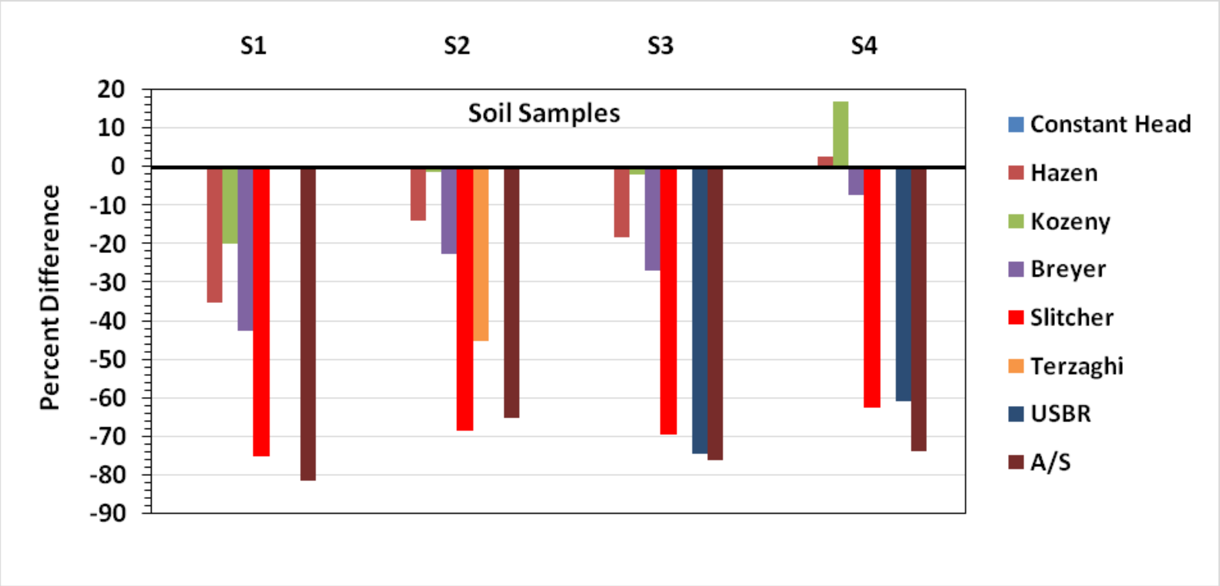


Figure 2: Comparative results of Hydraulic conductivity of empirical formulae and constant head permeameter

Overall results showed that the hydraulic conductivities calculated by the Alyamani & Sen, USBR and Slitcher methods are in all cases lower than from the other methods as well as from constant head test results. And also according to (Vukovic and Soro 1992) and (Cheng and Chen 2007), these methods are always considered inaccurate. Likewise, Terzaghi method gave similar low values, may be due to the use of an average value of sorting coefficient in the formula. Breyer method is most accurate as compared to the other above four methods, but the results are underestimated than constant head test for all samples. The results of sample 2 and 4 were 22.74% and 7.35 % lower than constant head method respectively. Hazen formula which is based only on the d10 particle size is less accurate than the Kozeny-Carman formula which is based on the entire particle size distribution and particle shape (Carrier 2003). Therefore, the estimations by Kozeny- Carman for samples (1, 2, &3) were more accurate than hazen, and possibly the best estimations in this study and others. Kozeny-Carman however, underestimated sample1 (19.98%) more as compared with other samples (2&3) since the formula is not appropriate if the particle distribution has a long, flat tail in the fine fraction (Carrier 2003).The hazen formula gave the best results for sample 4 but overestimated (2.80%) as compared to the K-C equation which gave (17.06%) overestimated results. Alyamani and Sen Method is very sensitive to the shape of the grading curve and is more accurate for well graded samples. Consequently, it gave underestimated values of all samples due to their poor grading.

4. CONCLUSIONS

Based on the aforementioned analysis and results, the following conclusions can be drawn:

(1) Estimating the hydraulic conductivity of soils in terms of grading characteristics can relatively lead to underestimation or overestimation unless the appropriate method is used.

(2) For the studied samples, the best overall estimation of permeability is reached based on Kozeny-Carman's formula followed by Hazen formula and then Breyer formula.

(3) Slitcher, USBR, Terzaghi and Alyamani & Sen formulae grossly underestimated the hydraulic conductivities in comparison to the other evaluated formulae. Alyamani and Sen Formula are very sensitive to the shape of the grading curve.

(4) In this study the most suitable formulae for estimation of hydraulic conductivities were as follows.
  - Sample 1 (Kozeny-Carman): 24.26 m/day; with Hazen formula acceptable
  - Sample 2 (Kozeny-Carman): 117.89 m/day
  - Sample 3(Kozeny-Carman): 70.56 m/day with Hazen and Breyer formulae acceptable
  - Sample 4 (Hazen and Breyer): 57.35 m/day and 51.69 m/day

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